Non-perturbative Implications of Hadronic Interactions

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We consider the vacuum structure of an effective theory of rho mesons, pions and electromagnetism. The implications are hadronic analogues of the sphaleron and electroweak strings, and an associated primordial magnetogenesis mechanism. Initial estimates suggest about five thousand times more magnetic flux than a first order electroweak phase transition.

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1. Introduction.— In a related paper [1] we proposed a gauge theoretic description of the effective hadronic interactions of rho mesons and pions. The resultant theory is similar to the Weinberg-Salam model, and naturally incorporates electromagnetism. Within its framework the rho mesons may be thought of as counterparts of the W and Z gauge bosons, whilst the pion and sigma are associated with the Higgs field. Electromagnetism is considered similarly in both cases and represents a residual, unbroken gauge symmetry.

Predictions of this theory give a good description of the decay of the rho meson and the pion-pion scattering amplitudes. We found agreement for the rho meson decay widths, and the pion-pion phase shifts appear consistent up to about a GeV or so. Above a GeV effects from other particles become important.

In this letter we further discuss this similarity between electroweak symmetry breaking and the hadronic interactions of the pions and rho meson. In particular we discuss the consequences of the associated non-trivial vacuum structure.

To be more specific consider a complex pseudoscalar doublet composed of the sigma and pions fields

$$\Phi = \tfrac{1}{2}\mathbf{1}_2\sigma + \tfrac{1}{2}\pi_a\sigma_a, \tag{1}$$

the π_3 component is associated with the neutral pion, whereas the π_1 and π_2 components are associated with the charged pions

$$\pi = \frac{\pi_1 + i\pi_2}{\sqrt{2}}, \qquad \pi^\dagger = \frac{\pi_1 - i\pi_2}{\sqrt{2}}. \tag{2}$$

Also consider an $su(2)$-valued gauge field composed of rho meson fields

$$\rho^\mu = \rho_i^\mu \cdot \tfrac{1}{2}i\sigma_i. \tag{3}$$

It is necessary to also consider a $U(1)$ gauge field B^μ. It transpires that electromagnetism is a linear combination of ρ_3^μ and B^μ, as in the Weinberg-Salam model.

Then the effective theory of rho mesons, pions and electromagnetism is described by the following Lagrangian

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}\mathrm{tr}R^{\mu\nu}R_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + \tfrac{1}{2}\mathcal{D}^\mu\Phi^\dagger\mathcal{D}_\mu\Phi \\ &-\tfrac{1}{4}\lambda(\Phi^\dagger\Phi - f_\pi^2)^2,\end{aligned} \tag{4}$$

where the gauge field tensors are

$$R^{\mu\nu} = \partial^\nu\rho^\mu - \partial^\mu\rho^\nu + \tilde{g}[\rho^\mu,\rho^\nu], \tag{5}$$

$$B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu, \tag{6}$$

and the covariant derivative is

$$\mathcal{D}^\mu = \partial^\mu + \tfrac{1}{2}i\tilde{e}B^\mu + \tfrac{1}{2}i\tilde{g}\rho_i^\mu\sigma_i. \tag{7}$$

The above theory is invariant under an $SU(2)\times U(1)$ gauge symmetry associated with the ρ^μ and B^μ gauge fields. This is broken to a residual electromagnetic gauge symmetry

$$SU(2)\times U(1) \to U(1)_Q. \tag{8}$$

To exhibit the residual theory the vector meson and photon basis is rotated to a basis of mass eigenstates

$$\begin{pmatrix}\rho^{0\mu}\\ A^\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_s & -\sin\theta_s\\ \sin\theta_s & \cos\theta_s\end{pmatrix}\begin{pmatrix}\rho_3^\mu\\ B^\mu\end{pmatrix}, \tag{9}$$

associated with the corresponding generators

$$\begin{pmatrix}\tfrac{1}{2}\alpha X_0\\ eX_Q\end{pmatrix} = \begin{pmatrix}\cos\theta_s & -\sin\theta_s\\ \sin\theta_s & \cos\theta_s\end{pmatrix}\begin{pmatrix}\tfrac{1}{2}i\tilde{g}\sigma_3\\ \tfrac{1}{2}i\tilde{e}\mathbf{1}_2\end{pmatrix}. \tag{10}$$

Here θ_s we call the 'strong mixing angle', such that $\tan\theta_s = \tilde{e}/\tilde{g}$. It is also useful to express the rho meson gauge bosons in a charge eigenstate basis

$$\rho^\mu = \frac{\rho_1^\mu + i\rho_2^\mu}{\sqrt{2}}, \qquad \rho^{\dagger\mu} = \frac{\rho_1^\mu - i\rho_2^\mu}{\sqrt{2}}, \tag{11}$$

with the charge neutral component ρ^0 defined in Eq. (9).

Because of hadronic processes the pion mass is nonzero and must be included by hand. This leads to a partial symmetry breaking effect, whereby the Goldstone bosons are not actually massless:

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}\mathrm{tr}R^{\mu\nu}R_{\mu\nu} - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + \tfrac{1}{2}\partial^\mu\pi^0\partial_\mu\pi^0 + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma \\ &+ D^\mu\pi^\dagger D_\mu\pi + \tfrac{1}{2}m_\pi^2(2\pi^\dagger\pi + \pi^0\pi^0) + m_\rho^2\rho_\mu^\dagger\rho^\mu \\ &+\tfrac{1}{2}m_{\rho_0}^2\rho_\mu^0\rho^{0\mu} + \tfrac{1}{2}m_\sigma^2\sigma\sigma + \cdots\end{aligned} \tag{12}$$

with the residual covariant derivative

$$D^\mu = \partial^\mu + ieA^\mu. \tag{13}$$

Experimentally $m_\pi^2 \sim 140\,\mathrm{MeV}$, $f_\pi \sim 92\,\mathrm{MeV}$ and $m_\rho \sim 770\,\mathrm{MeV}$. From [1] we take $\tilde{g} = 12.2$, implying $\sin^2\theta_s \sim 6\times 10^{-4}$. Since m_π is non-zero, but considerably less that m_ρ, the pion is interpreted as an approximate Goldstone boson.

The point of this letter is that the above reasoning suggests that associated with the quark-hadron phase transition is a vacuum manifold

$$M \cong \frac{SU(2)\times U(1)}{U(1)_Q}. \tag{14}$$

This results in a spectrum of non-perturbative solutions and effects similar to those found in electroweak theory.

An important approximation that we shall make is to set the mass terms m_π^2 and m_0^2 to zero. We shall take this as a reasonable first approximation. Inclusion of the masses would considerably complicate the following analysis. Furthermore results from [2] strongly suggest that the geometric properties of the vacuum determine the non-perturbative effects. These geometric properties are unaffected by the mass terms. However the validity and effects of this approximation should certainly be considered in future work.

The specific non-perturbative consequences we discuss are analogues of electroweak strings and the sphaleron.In addition we discuss a magnetogenesis mechanism.

2. Hadronic-Strings.— In electroweak theory there are string solutions corresponding to Nielson-Oleson vortices embedded within Weinberg-Salam theory [3]. The resultant hadronic counterparts are rho strings. They have similar solutions, with the ρ^0-string analogous to the Z-string

$$\sigma(r,\theta) = f_\pi f_{\mathrm{NO}}(r)\cos\theta, \tag{15}$$
$$\pi^0(r,\theta) = f_\pi f_{\mathrm{NO}}(r)\sin\theta, \tag{16}$$
$$\underline{\rho}^0(r,\theta) = \frac{g_{\mathrm{NO}}(r)}{r}\underline{\hat{\theta}}, \tag{17}$$

and the ρ^1, ρ^2 strings analogous to W-strings

$$\sigma(r,\theta) = f_\pi f_{\mathrm{NO}}(r)\cos\theta, \tag{18}$$
$$\pi^i(r,\theta) = f_\pi f_{\mathrm{NO}}(r)\sin\theta, \tag{19}$$
$$\underline{\rho}^i(r,\theta) = \frac{g_{\mathrm{NO}}(r)}{r}\underline{\hat{\theta}}. \tag{20}$$

The ρ^0-string may be expected to be stable for a similar range of parameters as the Z-string, namely when θ_s is close to $\pi/2$. More precisely, when $\theta_s \to \pi/2$ so that $\tilde{g} \to 0$, the strong isospin electromagnetic symmetry breaking of Eq. (8) becomes the semilocal model [4]

$$SU(2)_{\mathrm{global}} \times U(1) \to U(1)_Q, \tag{21}$$

with $U(1)$ a local symmetry and $U(1)_Q$ global. This theory admits dynamically stable semi-local vortex solutions providing that

$$\frac{2\lambda}{\tilde{e}^2} < 1. \tag{22}$$

By continuity the ρ^0 string is stable for a finite region of parameter space close to $\theta_s = \pi/2$. This region has been calculated for electroweak theory and is found to extend to about $\theta_s \sim 0.9\pi/2$ [5]. This is well outside the region of physical validity, both in θ_s and for λ.

However there will also be further stabilising effects. For electroweak theory this issue is unresolved, and it may be possible that fermion zero modes stabilise the Z-string [6]. For the ρ^0-string the analogous effect would be from nucleon zero modes on the string.

We should mention that the rho-strings are similar to pion strings [7]. However pion strings are global embedded defects, whilst rho-strings are local. This feature is crucial for their stability.

3. Magnetic Monopoles.— Nambu has considered an electroweak configuration that asymptotically represents a magnetic monopole [8]. Because of the geometry of electroweak theory, the configuration actually represents a non-topological monopole on the terminus of a Z-string.

The resultant hadronic counterparts are magnetic monopoles on the end of ρ^0-strings.

A qualitative argument for their existence may be given as follows [9]. A ρ^0-string is non-topological, and thus may terminate. Since the B component of the ρ^0 field in the string is divergenceless, it must continue beyond the end of the string. However the B flux is massive, and by energetic arguments may not continue. The only means by which it may continue is in combination with the $SU(2)$ gauge field as a massless electromagnetic flux. It does this via a electric monopole.

The amount of magnetic flux emanating from the end of the ρ^0-string is estimated using an analogous expression to that from Z-string. The flux is

$$F = \frac{4\pi}{e}\sin^2\theta_s. \tag{23}$$

about three orders of magnitude smaller than that of electroweak monopoles.

For more details of such configurations, in the context of electroweak theory, we refer to [9].

4. Hadronic-Sphalerons.— The counterpart of the electroweak sphaleron [10] is the configuration

$$\Phi(\mathbf{r}) = f_{\mathrm{sph}}(r)\exp\Big(\frac{i\pi}{2}\hat{r}_a\sigma_a\Big)\Phi_0, \tag{24}$$

$$\rho_a(\mathbf{r}) = g_{\mathrm{sph}}(r)\frac{i}{2}\epsilon_{abc}\hat{r}_b\sigma_c, \tag{25}$$

representing a solution when $\theta_s = 0$. For values of the strong mixing angle close to zero the solution deforms from the above Ansatz inducing, to lowest order, a dipole moment in the electromagnetic field

$$\delta Q_a = \frac{\epsilon_{abc}\mu_b\hat{r}_c}{4\pi r^3}X_Q, \tag{26}$$

with $\underline{\mu}$ parallel to $\underline{\hat{r}}^3$, as found by substitution into the field equations.

In electroweak theory the importance of the sphaleron is its baryon violating effects. This arises through the coupling of the electroweak gauge fields to the standard model fermions. The counterpart here is the nucleon doublet $N = (n, p)$. Their natural rho interaction is to couple fundamentally to the rho gauge field. Hence their effects may be included through the Lagrangian

$$\mathcal{L}_N = i\bar{N}\gamma_\mu \mathcal{D}^\mu N + m_N^2 \bar{N}N, \tag{27}$$

with $\mathcal{D}^\mu$ the covariant derivative of Eq. (7), and $m_N \sim 1$ GeV. However, unlike electroweak theory, it transpires that the associated nucleon current

$$j_N^\mu = \tfrac{1}{2}\bar{N}\gamma^\mu(1-\gamma_5)N \tag{28}$$

is not anomalous, and does not couple to the sphaleron transitions.

This is seen as follows. Although $\mathcal{L}_N$ is invariant under a global $U(1)$ axial transformation

$$N \rightarrow N' = e^{-i\theta\gamma_5}N \tag{29}$$

and hence has anomalous axial current $\bar{N}\gamma^\mu\gamma_5 N$, the nucleon current does not contain an axial component because the N_L and N_R chiral components couple equally to the $SU(2) \times U(1)$ gauge fields. Hence, although rho-sphaleron transitions are related to an anomalous current, they do not give baryon number violation.

The above result is maybe unsurprising when one appreciates that the standard model preserves baryon number minus lepton number. There does not seem to be a method of violating lepton number with the hadronic sphaleron, and hence baryon number should also not be violated.

5. Cosmological Magnetogenesis— The electroweak phase transition has been suggested as a reasonable way of producing a primordial magnetic flux. The mechanisms rely on the uncorrelated phases of the vacuum after symmetry breaking [11]. In particular, bubble collisions from a first order electroweak phase transition seem to be successful way of generating a primordial magnetic flux. They produce roughly the right amount to seed the galactic magnetic field [12,13].

Since lattice simulations of the quark-hadron phase transition show it to be first order [14], and we are considering an effective Weinberg-Salam theory of hadrons, it is a prudent question to enquire as to whether an analogous mechanism could apply to create magnetic flux at the quark-hadron phase transition.

This is actually quite a difficult question to answer. The production of magnetic flux depends upon the detailed dynamics of the bubble collisions. Hence, as a first step, we simply make an order of magnitude estimate of the comparative magnetic fluxes based upon results from electroweak theory. They compare as

$$\frac{B_{\rm QCD}}{B_{\rm ewk}} = n_1 n_2 n_3, \tag{30}$$

where the three factors are:
(i) The initial magnitude of magnetic field produced at the phase transition. For bubble collisions in electroweak theory this is proportional to $\sin^3\theta_w/g$, where g is the weak isospin [13]. This gives rise to a relative scale

$$n_1 = \frac{g\sin^3\theta_s}{\tilde{g}\sin^3\theta_w} \sim 5\times 10^{-5}. \tag{31}$$

taking $\sin^2\theta_w \sim 0.22$ and $g \sim 0.65$.
(ii) The expansion of the universe between the electroweak scale and the quark-hadron scale. Magnetic flux scales as area, leading to

$$n_2 = \frac{R_{\rm QCD}^2}{R_{\rm ewk}^2} = \frac{T_{\rm ewk}^2}{T_{\rm QCD}^2} \sim 10^6, \tag{32}$$

in the radiation era. Here R is the scale factor and T is the temperature.
(iii) The relative sizes of the bubbles formed, related to the correlation lengths that compare as

$$\frac{\xi_{\rm ewk}}{\xi_{\rm QCD}} \sim \frac{m_\rho}{m_W} \sim 10^{-2}. \tag{33}$$

This results in a comparative volume averaging. There has been some dispute in the relevant average to take. We will follow [15], using their volume average factor

$$n_3 = \left(\frac{m_\rho}{m_W}\right)^{-3/2} \sim 10^3. \tag{34}$$

An area average will result in $n_3 \sim 10^2$.

From the above we have

$$\frac{B_{\rm QCD}}{B_{\rm ewk}} \sim 5\times 10^3. \tag{35}$$

Thus it appears that the quark-hadron phase transition is capable of producing considerably more magnetic flux than a first order electroweak phase transition.

As a final note we should mention that current bounds on the Higgs mass imply that the Weinberg-Salam model does not give a first order phase transition.

6. Conclusions— By using the effective theory of [1], which gives a phenomenologically successful description of many hadronic interactions, we claim that the quark-hadron phase transition results in many non-perturbative effects that are analogous to electroweak theory. In particular it gives rise to both hadronic string solutions and sphalerons. Unlike electroweak theory the rho-sphalerons do not couple to baryon number, and hence may not be used for a baryogenesis mechanism.

We also tentatively propose that the quark-hadron phase transition may give rise to a residual magnetic flux, created by bubble collisions.

Some further issues are:

(i) Most importantly the issue of considering a non-zero m_π^2 and m_0^2 should be addressed. Inclusion will effect the profiles and energetics of the hadronic strings and sphaleron. Also it will effect the features of hadronic bubble-bubble collisions, important in the production of magnetic flux. However, as said before, we do not expect it to affect the quantitative features of the above analysis.

(ii) As well of being of possible cosmological significance the above effects could also, in the context of our effective theory, be produced in disordered chiral condensates.

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[1] N.F.Lepora, hep-ph/9910517.

[2] N.F. Lepora and T.W.B.Kibble, JHEP **04** (1999) 027 hep-th/9904178.

[3] T. Vachaspati, Phys. Rev. Lett. **68** (1992) 1977, *ibid* **69**, 216 (1992). T.Vachaspati and M.Barriola, Phys. Rev. Lett. **69** (1992) 1867. N. Lepora and A. C. Davis, Phys. Rev. **D58** (1998) 125027 hep-ph/9507466.

[4] T. Vachaspati and A. Achúcarro, Phys. Rev. D **D44** (1991) 3067. M. Hindmarsh, Phys. Rev. Lett. **68** (1992) 1263.

[5] M. James, L. Perivolaropoulos and T. Vachaspati, Phys. Rev. D **46** (1992) 5232.

[6] M.Groves and W.Perkins, hep-ph/9908416.

[7] X. Zhang, T. Huang and R.H. Brandenberger, Phys. Rev. **D58** (1998) 027702 hep-ph/9711452.

[8] Y. Nambu, Nucl. Phys. **B130** (1977) 505.

[9] A. Achucarro and T. Vachaspati, hep-ph/9904229.

[10] N. Manton, Phys. Rev. D **28**, 2019 (1983). F. Klinkhamer and N. Manton, Phys. Rev. D **30**, 2212 (1984).

[11] T. Vachaspati, Phys. Lett. **B265** (1991) 258.

[12] T.W.B.Kibble and A. Vilenkin, Phys. Rev. **D52** (1995) 679 hep-ph/9501266. J. Ahonen and K. Enqvist, Phys. Rev. **D57** (1998) 664 hep-ph/9704334. A. Davis, M. Lilley and O. Tornkvist, Phys. Rev. **D60** (1999) 021301 astro-ph/9904022.

[13] O.Tornkvist, hep-ph/9902432.

[14] K. Kanaya, Nucl. Phys. Proc. Suppl. **47** (1996) 144 hep-lat/9510040.

[15] M. Hindmarsh and A. Everett, Phys. Rev. **D58** (1998) 103505 astro-ph/9708004.